change is small and the shock can be treated as a simple wave to which either eq. (35) or (36) applies. In this approximation the interactions of shock waves and rarefactions can be calculated from eqs. (25) and (26).

3. – Elementary wave interactions.

Equations (32), (34), (35) and (36) uniquely define and limit the values of particle velocity, u, which can be achieved by simple shock or rarefaction from a given state (p_0, V_0, u_0). This limitation on states which can be reached in a single wave transition supplies a powerful tool for thinking about and calculating the fields of high-amplitude waves. The problem is transformed into a «hodograph» plane in which the variables are (u, p), (u, l), (r, s) or some equivalent set. We shall use u, p here because of continuity conditions on u and p at an interface or boundary. The significance of this choice will appear later.

Various useful representations of a shock and of a rarefaction are shown in Fig. 4. In 4 *a*) is a cross-section of a half-space to which a pressure p_1 was applied at $t=0$ and released at $t=t_0$. The pressure profile at this particular $t>t_0$ is shown in 4 *b*). It consists of a forward-facing shock, designated $\mathscr{S}_+$, a region of uniform pressure p_1 and particle velocity u_1, and a rarefaction $\mathscr{R}_+$. The notations $\mathscr{S}$ and $\mathscr{R}$ are introduced here to denote shock and rarefaction waves, respectively. Forward-facing waves are denoted by the subscript «$+$», backward-facing by «$-$». In Fig. 4 *c*) the flow is shown in the (x, t) plane. Region I is the uniform initial state (p_0, V_0, u_0) with $u_0>0$. The shock front, $\mathscr{S}_+$, has constant slope until the following rarefaction overtakes it, reducing its amplitude and velocity. Region II is the uniform state (p_1, u_1, V_1) behind the shock. Region III is the rarefaction $\mathscr{R}_+$ in which pressure and particle velocity are diminishing. Region IV is again at the ambient pressure p_0 but volume and particle velocity now differ from V_0 and u_0. The path OAB is the trace of the half-space surface, sometimes called the «piston path», $\mathscr{P}$. The dashed curve is the path of a single particle or mass element traversed successively by $\mathscr{S}_+$ and $\mathscr{R}_+$. Figure 4 *d*) shows the wave process in the (p, V) plane. The initial shock compression is along the Rayleigh line to the state B on the Hugoniot. The rarefaction, assumed to be isentropic, expands the material along the dashed isentrope to the final state $C(V_0', p_0, u_0')$. In Fig. 4 *e*) the process is shown in the (p, u) plane. The straight line AB with slope $\mathrm{d}p/\mathrm{d}u=\varrho_0(D-u_0)$ is the image of the Rayleigh line. The compressed state B lies on the image of the Hugoniot curve and the dashed curve BC is the image of the isentrope of Fig. 4 *d*). Because the shock process is entropic and because most materials have positive thermal expansion coefficients, the final state (u_0', p_0) is normally to the left of (u_0, p_0) for forward-facing waves.

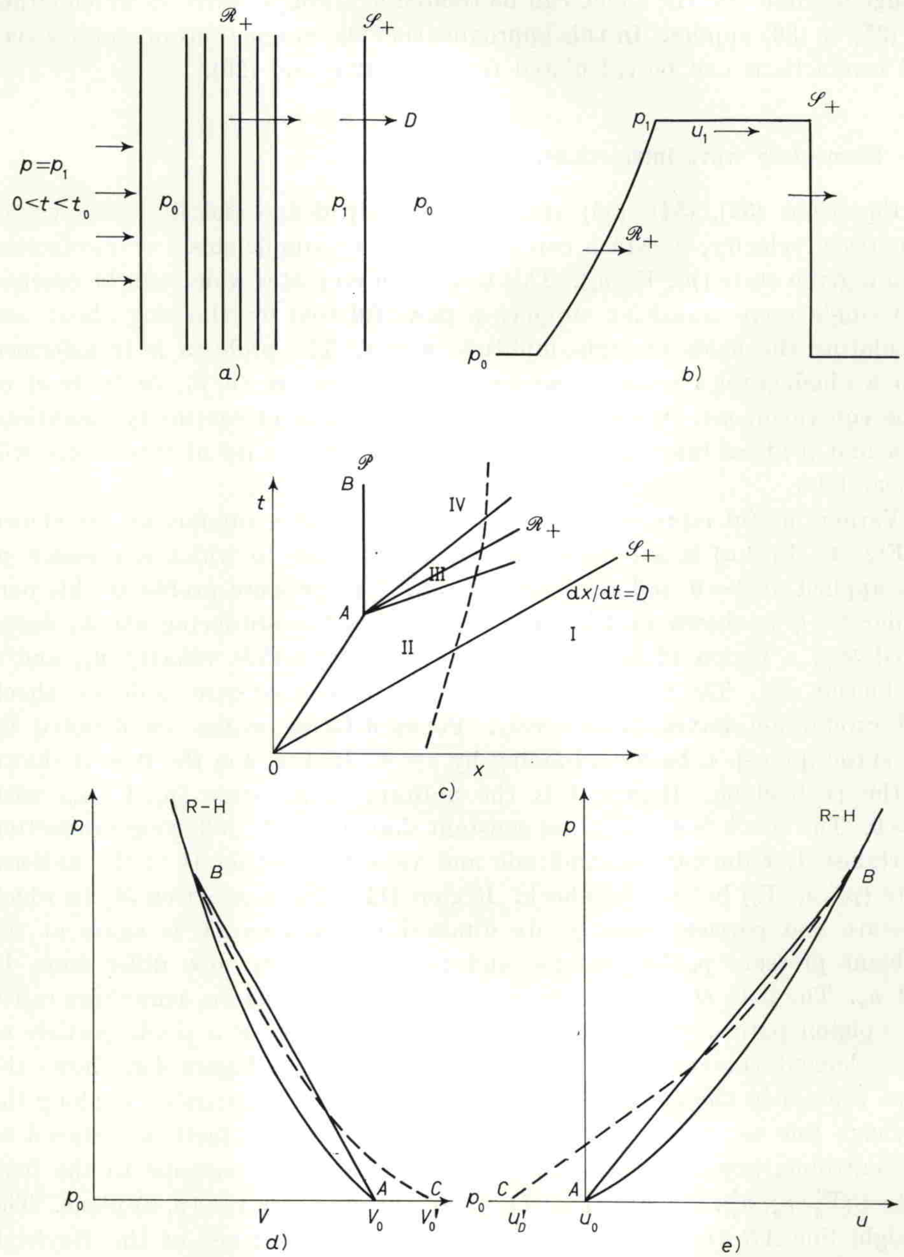

Fig. 4. – Forward-facing rarefaction overtaking a shock. *a*) planes of constant phase in half-space; *b*) pressure profile, $t > t_0$; *c*) $(x\text{-}t)$ diagram; *d*) $(p\text{-}V)$ diagram; *e*) $(p\ u)$-plane.